Grade 1 Mathematics, Quarter 1, Unit 1.1

Counting and Representing Numbers Within 60

Overview

Number of instructional days: 8
(1 day = 45-60 minutes of instruction)

Content to be learned

- Count orally to 60 starting at any number less than 60.
- Read and write numerals to 60.
- Represent up to 60 objects with a numeral.

Essential questions

- What tools can you use to count to 60?
- How do you know which number comes next when you are counting to 60?
- When given a sequence of numbers, how do you know if the sequence is correct or incorrect?
- What strategies can you use to count a quantity of objects up to 60?

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Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities up to 60 and represent it symbolically.
- Attend to the meaning of quantities.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	1.NBT

Extend the counting sequence.

1.NBT.1 Count ~~to 120~~, starting at any number ~~less than 120~~. ~~In this range~~, read and write numerals and represent a number of objects with a written numeral.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students rote counted sequentially to 100 by ones and tens. They wrote numbers up to 20. Students represented a number of objects (0–20) with a written numeral.

Current Learning

Students count to 60 starting at any number less than 60. They are writing and representing a number of up to 60 objects with a written numeral. This is a major cluster, which means greater emphasis should be placed on this skill, because it sets a foundation for future learning. Building on the prior knowledge obtained in kindergarten, the instructional level of this unit is developmental.

Future Learning

In second grade, students will count within 1,000 as well as skip-count by 5s, 10s, and 100s. They will read and write numbers to 1,000.

Additional Findings

According to *Principles and Standards for School Mathematics*, emphasis should be placed on developing concepts of time and the way it is measured. Throughout the school day, opportunities arise for teachers to focus on time through short conversations with their students. Students will learn to tell time as attention is called to the clock by the teacher (p. 104).

Grade 1 Mathematics, Quarter 1, Unit 1.2

Adding Within 20

• Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Demonstrate fluency for addition facts within 10.
- Use the strategy of counting on when adding.
- Understand the meaning of the equal sign.
- Determine if addition equations within 20 are true or false.
- Count on by two to add 2 when solving addition problems.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Plan how to solve the problem by counting on.
- Use concrete objects (manipulatives) or pictures to help conceptualize and solve a problem.

Reason abstractly and quantitatively.

- Use symbolic representations to move from a concrete idea to a written equation.
- Stop and self-check throughout the process to determine accuracy of the outcome.

Model with mathematics.

- Identify important quantities when counting on.
- Analyze addition equations to draw conclusions as to the accuracy of the solution of an equation.

Essential questions

- What does it mean to add?
- How can you use the strategy of "counting on" when adding?
- What does the equal sign mean?
- How can you solve the problem quickly?
- When writing a number sequence, how do you know what number comes next?

• Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Add and subtract within 20.

1.OA.6 Add ~~and subtract~~ within 20, demonstrating fluency for addition ~~and subtraction~~ within 10. Use strategies such as counting on; ~~making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).~~

Work with addition and subtraction equations.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition ~~and subtraction~~ are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Add and subtract within 20.

1.OA.5 Relate counting to addition ~~and subtraction~~ (e.g., by counting on 2 to add 2).

*1.OA.7 Students should have an understanding of the equal sign when interviewed in small groups or individually.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students represented addition with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. Students were encouraged to write equations, but it was not required. They added within 10 by using objects, drawing, or equations to represent the problem. They found the number that makes 10 when added to any given number from 1–9. Fluency was gained for adding within 5.

Current Learning

Students add within 10 by counting on from a given number. They recognize and correctly use the addition sign and the equal sign. Fluency in addition within 10 is expected by the end of first grade. The instructional level for this unit is developmental. This is a major cluster, which requires that greater emphasis be placed on this skill.

Future Learning

In second grade students, will be expected to use addition and subtraction within 100 to solve one- and two-step word problems. They will fluently add within 20 using mental strategies and know from memory all sums of two one-digit numbers.

Additional Findings

As stated in *Adding It Up*, "They notice that they do not have to count the objects for the first addend but can start with a number in the first or the larger addend and count on the objects in the other addend (count on)." (p. 187)

As stated in *Curriculum Focal Points*, students create strategies for solving problems based on their understanding of the meanings of numbers. (p. 11)

Grade 1 Mathematics, Quarter 1, Unit 1.3

Finding the Sum in Word Problems

• Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve addition word problems within 20 by using drawings, objects, and equations.
- Solve various word problem types, including adding to and putting together with unknowns in the sum position.
- Solve addition problems using the strategies counting on and putting together.
- Use equations with a symbol for the unknown to represent the problem being solved.
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.
- Add within 20 while working towards fluency within 10.
- Understand of the meaning of the equal sign.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Demonstrate understanding by explaining the strategy used to solve the problem.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Write equations to describe a situation.
- Identify important quantities in addition situations.
- Represent word problems using pictures and/or manipulatives.

Essential questions

- How can you check your work to be certain it is correct?
- How do you know when to add to solve a problem?
- What order do the addends have to be in to find the correct sum?
- What is your strategy for solving this word problem?
- What strategies can you use to solve these equations?
- How does your drawing (or manipulative) represent the problem? How does your drawing (or manipulative) help you solve the problem?

• Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and ~~subtraction~~ within 20 to solve word problems involving situations of adding to, ~~taking from~~, putting together, ~~taking apart, and comparing,~~ with unknowns in ~~all~~ positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

1. OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Work with addition and subtraction equations.

1. OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and ~~subtraction~~ are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

*1.OA.1- Mastery is expected in "Add To" ~~and "Take From" -~~ Result ~~and Change Unknown Problems~~, "Put Together/~~Take Apart~~" Problems

1.OA.2 Interviews (individual or small group) should target student's ability to solve word problems with three addends.

1.OA.7 Students should have an understanding of the equal sign when interviewed in small groups or individually.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In Kindergarten, students represented addition with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. They solved addition word problems and added within 10. By the end of Kindergarten, students had achieved fluency in adding within 5. In Quarter 1 of first grade, students added within 10.

Current Learning

Students add within 20 to solve word problems using various strategies. They solve word problems involving three whole numbers whose sum is less than or equal to 20. They write equations with a symbol for the unknown number in the sum position to represent the problems. They understand the meaning of the equal sign. They are working toward fluency in addition within 10. This is a critical area and involves developmental as well as reinforcement activities. (In Standard 1.OA.1, notice the footnote, "See Glossary, Table 1.")

Future Learning

In second grade students will be expected to use addition within 100 to solve one- and two-step word problems. They will fluently add within 20 using mental strategies and know from memory all sums of two one-digit numbers.

Additional Findings

According to *Adding It Up*, "Young children are able to make sense of the relationships between quantities and to come up with appropriate counting strategies when asked to solve simple word or story problems. Word problems are often thought to be more difficult than simple number sentences or equations. Young children, however, find them easier" (p. 169).

"Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers" (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics,* p. 13).

Grade 1 Mathematics, Quarter 1, Unit 1.4

Introducing Place Value

• Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand that amounts of tens and ones are represented by two digits.
- Understand that 10 ones can be called a "ten."

Mathematical practices to be integrated

Attend to precision.

- Attend to the meaning of quantities.
- Clearly define that a bundle of ten ones is a

- Develop understanding of the relative position of whole numbers.
- Understand numbers 11 to 19 are composed of a “ten” and one, two, three, four, five, six, seven, eight, or nine “ones.”

“ten.”

Look for and express regularity in repeated reasoning.

- Notice that is one “ten” is added, the numeral in the tens place increases by one, but adds a quantity of 10.

Essential questions

- How can you represent the tens and ones in a two-digit number?
- How do you know the number of “tens” in a given number?
- When is the appropriate time to use the term “ten”?
- When looking at a number from 11 to 19, why does the first number stay the same, while the second number changes?

• Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**1.NBT**

Understand place value.

1. NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a “ten.”

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students composed and decomposed numbers from 11–19 into ten ones and some further ones. They grasped the understanding that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones. Students were not exposed to the term “a ten” for a bundle of ten ones; instead they simply called it “ten ones.”

Current Learning

Students identify a bundle of 10 ones as a “ten.” Students understand that the numbers from 11–19 are composed of a “ten” and one, two, three, four, five, six, seven, eight, or nine ones. Place value is considered a major cluster, therefore greater emphasis should be placed on this content. Due to the prior knowledge built in kindergarten, the instructional level of this unit is developmental.

Future Learning

In second grade, students will understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. The will think of 100 as a bundle of ten tens and call it a “hundred.”

Additional Findings

As stated in *Adding It Up*, “The base ten place value system is very efficient. It allows one to write very large numbers using only ten symbols, the digits 0–9. The same digit has a different meaning depending on its place in the numeral.” (p. 199)

As stated in *Principles and Standards for School Mathematics*, “During the early years, teachers must help students strengthen their sense of number, moving from the initial development of basic counting techniques to more sophisticated understanding of the size of numbers, number relationships, patterns, operations, and place value.” (p. 79)

Grade 1 Mathematics, Quarter 2, Unit 2.1

Counting and Representing Numbers Within 120

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Count orally to 120 starting at any number less than 120.
- Read and write numerals to 120.
- Represent up to 120 objects with a numeral.
- Understand that amounts of tens and ones are represented by two digits.
- Understand that 10 ones can be called a "ten."
- Develop understanding of the relative position of whole numbers.
- Understand numbers 11 to 19 are composed of a "ten" and one, two, three, four, five, six, seven, eight, or nine "ones."

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities up to 120 and represent it symbolically.
- Attend to the meaning of quantities.
- Clearly define that a bundle of ten ones is a "ten."

Essential questions

- What tools can you use to count to 120?
- How do you know which number comes next when you are counting to 120?
- How can you represent the tens and ones in a two-digit number?
- How do you know the number of "tens" in a given number?
- When is the appropriate time to use the term "ten"?
- When given a sequence of numbers, how do you know if the sequence is correct or incorrect?
- What strategies can you use to count a quantity of objects up to 120?
- When looking at a number from 11 to 19, why does the first number stay the same, while the second number changes?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**1.NBT**

Extend the counting sequence.

1. NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

1. NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students rote counted sequentially to 100 by ones and tens. They wrote numbers up to 20. Students represented a number of objects (0–20) with a written numeral. In kindergarten, students composed and decomposed numbers from 11–19 into ten ones and some further ones. They grasped the understanding that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones. Students were not exposed to the term "a ten" for a bundle of ten ones; instead they simply called it "ten ones."

Current Learning

Students count to 120 starting at any number less than 120. They are writing and representing a number of up to 120 objects with a written numeral. This is a major cluster, which means greater emphasis should be placed on this skill, because it sets a foundation for future learning. Building on the prior knowledge obtained in kindergarten, the instructional level of this unit is developmental. Students identify a bundle of 10 ones as a "ten." Students understand that the numbers from 11–19 are composed of a "ten" and one, two, three, four, five, six, seven, eight, or nine ones.

Future Learning

In second grade, students will count within 1,000 as well as skip-count by 5s, 10s, and 100s. They will read and write numbers to 1,000. In second grade, students will understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. The will think of 100 as a bundle of ten tens and call it a "hundred." They will understand that the numbers 100, 200, 300, etc. refer to one, two, or three hundreds and zero tens and zero ones.

Additional Findings

As stated in *Adding It Up*, a student's ability to count does not demonstrate a student's comprehension of number value. (p. 161)

In *Principles and Standards for School Mathematics*, research finds that "During the early years, teachers must help students strengthen their sense of number, moving from the initial development of basic counting techniques to more-sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value." (p. 79)

As stated in *Adding It Up*, "The base ten place value system is very efficient. It allows one to write very large numbers using only ten symbols, the digits 0–9. The same digit has a different meaning depending on its place in the numeral." (p. 199)

Grade 1 Mathematics, Quarter 2, Unit 2.2
Subtracting Within 20

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Demonstrate fluency for subtraction facts within 10.
- Use the strategy of counting back when subtracting.
- Understand the meaning of the equal sign.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Plan how to solve the problem by counting back.
- Use concrete objects (manipulatives) or pictures to help conceptualize and solve a problem.

Reason abstractly and quantitatively.

- Use symbolic representations to move from a concrete idea to a written equation.
- Stop and self-check throughout the process to determine accuracy of the outcome.

Model with mathematics.

- Identify important quantities when counting back.
- Analyze subtraction equations to draw conclusions as to the accuracy of the solution of an equation.

Essential questions

- What does it mean to subtract?
- How can you use the strategy of "counting back" when subtracting?
- What does the equal sign mean?
- How can you solve the problem quickly?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Add and subtract within 20.

1. OA.6 ~~Add and~~ subtract within 20, demonstrating fluency for ~~addition and~~ subtraction within 10. Use strategies such as counting back; ~~making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).~~

Work with addition and subtraction equations.

1. OA.7 Understand the meaning of the equal sign, and determine if equations involving ~~addition and~~ subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

*1.OA.7 Students should have an understanding of the equal sign when interviewed in small groups or individually

1. OA.5 Relate counting to ~~addition and~~ subtraction (e.g. by counting back by 2 to subtract 2)

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students represented subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. Students were encouraged to write equations, but it was not required. They subtracted within 10 by using objects, drawing, or equations to represent the problem. Fluency was gained for subtracting within 5.

Current Learning

Students subtract within 10 by counting on from a given number. They recognize and correctly use the minus sign and the equal sign. Fluency in subtraction within 10 is expected by the end of first grade. The instructional level for this unit is developmental. This is a major cluster, which requires that greater emphasis be placed on this skill.

Future Learning

In second grade students, will be expected to use addition and subtraction within 100 to solve one- and two-step word problems. They will subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Additional Findings

As stated in *Curriculum Focal Points*, students create strategies for solving problems based on their understanding of the meanings of numbers. (p. 11)

Grade 1 Mathematics, Quarter 2, Unit 2.3

Finding the Difference in Word Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve subtraction word problems within 10 using objects, drawings, and equations.
- Represent and solve problems using objects, drawings, and equations.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Act out and solve story problems.
- Breaking apart and using prior knowledge to solve problems.
- Abstract a given situation and represent it symbolically.

Model with mathematics.

- Use objects, drawings, or equations to model and solve subtraction situations.
- Analyze relationships to draw conclusions.
- Question if their prediction or solution makes sense.

Essential questions

- What strategies do you use to solve word problems?
- How do you know if this is an addition or subtraction problem?
- Can you explain the easiest and/or fastest way to solve this problem?
- How can you represent this problem using objects and equation(s)?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	1.OA

Represent and solve problems involving addition and subtraction.

1.OA.1 Use ~~addition and~~ subtraction within 20 to solve word problems involving situations of ~~adding to~~, taking from, ~~putting together~~, taking apart, and ~~comparing~~, with unknowns in ~~all~~ positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

1.OA.1- , Mastery is expected in “~~Add To~~” and “Take From”~~–~~ Result~~ and Change Unknown Problems~~, “~~Put Together/~~Take Apart” Problems

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In kindergarten, students solved addition and subtraction word problems, and added and subtracted within 10, e.g., by using objects or drawings to represent the problem. They used objects or drawings to record equations, e.g., $5 = 2 + 3$ and $5 = 4 + 1$). They fluently added and subtracted within 5. They also composed and decomposed numbers from 11 to 19 into ten ones and some further ones.

Current Learning

In first grade, students add and subtract within 20 to solve word problems involving situations using a variety of strategies. They apply properties of operations as strategies to add and subtract. Students do not need to use formal terms for these properties. They use a variety of strategies to add and subtract within 20 and demonstrate fluency within 10. They will also add within 100, including adding a two-digit number and a one-digit number using various strategies. This is a major cluster and will include introductory as well as developmental activities.

Future Learning

In second grade, students will be expected to use addition and subtraction within 100 to solve one- and two-step word problems. They will fluently subtract within 20 using mental strategies. They will know from memory all sums of two one-digit numbers.

Additional Findings

Word problems are often thought to be more difficult than simple numbers sentences or equations. Young children, however, find them easier. If the problems pose simple relationships and are phrased clearly, preschool and kindergarten children can solve word problems involving addition, subtraction, multiplication, or division. (*Adding It Up*, p. 169)

Leaning occurs best through dialogue, discussion, and interaction. Learners must be actively involved in the process. A variety of models must be used to meet the needs of all learners. Learners benefit from reviewing, critiquing and revising one another's work. (*A Research Companion to Principles and Standards for School Mathematics*, p. 48)

Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition. (*Progressions*, p. 360)

Grade 1 Mathematics, Quarter 2, Unit 2.4

Measuring Length with Nonstandard Units

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Order three objects by length.
- Compare the lengths of two objects indirectly by using a third object.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use appropriate tools.

- Measure an object by laying a nonstandard unit end to end with no gaps or overlaps.
- Express the length of an object as a whole number.
- Change the non-standard unit of measure if it does not help reach an accurate solution.
- Choose from a variety of mathematical tools that are readily available (Unifix® cubes, one-inch tiles, paper clips, popsicle sticks) to help them solve the problem.

Attend to precision.

- Accurately measure objects.

Essential questions

- How do know if one object is shorter or longer than another?
- How do you know which tools help measure length?
- What happens if there are gaps or overlaps between the units of measure?
- How can you find out which object is longer/shorter without placing them side-by-side?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**1.MD**

Measure lengths indirectly and by iterating length units.

1. MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1. MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

* Tasks should allow students to measure by laying physical units end-to- end. If students make procedural errors in measuring, they can be asked to tell in a precise way what the problem is, why it leads to incorrect measurements and how to fix it and measure more accurately.

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students described measurable attributes of objects, such as length. They directly compared two objects with a measurable attribute in common and described the difference.

Current Learning

In first grade, students order three objects by length and compare the length of two objects indirectly by using a third object. They express the length of an object using a variety of nonstandard units to span the length of the object with no gaps or overlaps. This is a major cluster and will consist of introductory activities.

Future Learning

In second grade, students will measure the length of the objects using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. They will measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Additional Findings

Children strengthen their sense of number by solving problems involving measurement and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. (*Curriculum focal Points for Prekindergarten through Grade 8 Mathematics,* p. 13)

Measurement is essential for developing an understanding of the natural world. By quantifying and otherwise mathematizing nature students can model the natural world, even at an early age. (*A Research Companion to Principles and Standards for School Mathematics*, p. 188)

Teachers should guide students' experiences by making the resources for measuring available, planning opportunities to measure, and encouraging students to explain the results of their actions. (*Principles and Standards for School Mathematics*, p. 103)

Grade 1 Mathematics, Quarter 3, Unit 3.1
Adding and Subtracting Within 20

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve addition and subtraction problems using various strategies such as counting on, making ten, and decomposing a number leading to a ten.
- Add and subtract within 20 while working towards fluency within 10.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Demonstrate understanding by explaining the strategy used to solve the problem.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Identify important quantities in addition and subtraction situations.

Essential questions

- How can you check your work to be certain it is correct?
- How do you know whether to use addition or subtraction to solve the problem?
- What order do the addends have to be in to find the correct sum?
- What strategies can you use to solve these equations?
- How does your drawing (or manipulative) represent the problem? How does your drawing (or manipulative) help you solve the problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Common Core State Standards for Mathematical Practice

Add and subtract within 20.

1. OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4);-and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Operations and Algebraic Thinking **1.OA**

Understand and apply properties of operations and the relationship between addition and subtraction.

1. OA.3 Apply properties of operations as strategies to add and subtract.[3] Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

[3] Students need not use formal terms for these properties.

Work with addition and subtraction equations.

1. OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 +2*

*1.OA.7 Students should have an understanding of the equal sign when interviewed in small groups or individually.

Clarifying the Standards

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Prior Learning

In Kindergarten, students represented addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. By the end of Kindergarten, students had achieved fluency in adding and subtracting within 5. In Quarter 1 of first grade, students added within 10.

Current Learning

Students add and subtract within 20 to solve equations using various strategies. They understand the meaning of the equal sign. They are working toward fluency in addition and subtraction within 10. This is a critical area and involves developmental as well as reinforcement activities. (In Standard 1.OA.1, notice the footnote, "See Glossary, Table 1.")

Future Learning

In second grade students will fluently add within 20 using mental strategies and know from memory all sums of two one-digit numbers.

Additional Findings

"Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers" (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics,* p. 13).

Grade 1 Mathematics, Quarter 3, Unit 3.2
True/False

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Determine whether an equation is true or false based on understanding of the meaning of the equal sign.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Demonstrate understanding by explaining the strategy used to solve the problem.
- Use concrete objects or pictures to help conceptualize and solve a problem.

Model with mathematics.

- Write equations to describe a situation.
- Identify important quantities in addition and subtraction situations.

Essential questions

- How can you check your work to be certain it is correct?
- How do you know if an addition or subtraction equation is true?
- What does the equal sign mean?
- How can you tell if an equation is true or false?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	1.OA

Work with addition and subtraction equations.

1. OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

*1.OA.7 Students should have an understanding of the equal sign when interviewed in small groups or individually.

Common Core State Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In Kindergarten, students represented addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. They solved addition and subtraction word problems and added within 10. By the end of Kindergarten, students had achieved fluency in adding and subtracting within 5. In Quarter 1 of first grade, students added within 10.

Current Learning

Students understand the meaning of the equal sign and determine whether addition and subtraction equations are true or false. This is a critical area and involves developmental as well as reinforcement activities. (In Standard 1.OA.1, notice the footnote, "See Glossary, Table 1.")

Future Learning

In second grade, students will be expected to compare three-digit numbers using

<, >, and = symbols to record the results of comparisons.

Grade 1 Mathematics, Quarter 3, Unit 3.3

Finding the Unknown Number in Equations

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Determine unknown whole number in an addition equation.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Represent the problem with visuals or

- Understand the relationship between three whole numbers in an addition equation.
- Determine the unknown number that makes the equation true.

manipulatives.

- Use prior knowledge to solve problems.
- Consistently think about how the problem and the solution fit together.

Look for and make use of structure.

- Students share strategies and different strategies with each other and discuss why different strategies provide the same correct answer.
- Look for connections between properties (e.g., 5 + 3 is the same as 3 + 5) and understand why it produces the same answer.

Look for and express regularity in repeated reasoning

- Discover connections between the procedure and the concept.

Essential questions

- How do you determine the unknown number in an addition or subtraction equation?
- After you determine the unknown number, how do you know if the equation is true?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Work with addition and subtraction equations.

1. OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.

Common Core State Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students solved an addition problem by adding any number to a given number to make 10.

Current Learning

In first grade, quarters 1 and 2, students add and subtract within 20. In this unit, students determine the unknown whole number in an addition equation, within 20, by relating three whole numbers. This is a major cluster and will include introductory activities.

Future Learning

In second grade, students will solve one- and two-step addition problems within 100 with unknowns in all positions.

Additional Findings

Students use properties of addition to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties to solve addition and subtraction problems involving basic facts. (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, p. 25)

Finding an unknown addend, (e.g., 8+__=14), one counts on from the known addend. The keeping track method is monitored so that counting on stops when the known total has been reached. The keeping track method tells the unknown addend. (*Progressions*, p. 37)

Grade 1 Mathematics, Quarter 3, Unit 3.4

Determining the Unknown Number in Word Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use the relationship between addition and subtraction to solve a subtraction problem.
- Understand that addition can be used to solve subtraction problems.
- Solve for the unknown in all positions in addition problems relating three whole numbers.
- Solve for the unknown in all positions in subtraction problems relating three whole numbers.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Think about and try several ways to solve the problem.
- Talk to other students about how they might solve the problem.
- Share thinking and solutions verbally and in writing.

Reason abstractly and quantitatively.

- Explain answers, not just how they arrived at them.
- Check for patterns or properties of operations they know how to solve.
- Use symbols to represent unknown numbers in the problem.

- Visualize what the problem is asking and use drawings to represent.

Essential questions

- What strategies can you use to solve for the unknown in a subtraction problem?
- How might visualizing the problem in your head help you solve the problem?
- How can addition help you solve a subtraction problem?
- What is your strategy for solving this problem?
- What is another way you can solve or think about this problem?
- How do you know your solution is correct?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart~~, and comparing,~~ with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

[2] See Glossary, Table 1.

1.OA.1- Mastery is expected in "Add To" and "Take From" - Result and Change Unknown Problems, "Put Together/Take Apart" Problems, "Compare

Understand and apply properties of operations and the relationship between addition and subtraction.

1. OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Work with addition and subtraction equations.

1. OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ☐ – 3, 6 + 6 = ☐.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students solved addition and subtraction problems within 10 by using objects, drawings, acted-out situations, verbal explanations, expressions, or equations to represent the problem.

Current Learning

In first grade, students work with addition and subtraction to solve problems. Students also solve for the unknown in addition situations. In quarter 3, students use the relationship between addition and subtraction to solve equations. Students now apply their knowledge of the relationship between addition and subtraction to solve for the unknown in all positions. *For example, 10 – 8 can be thought of as 10 is 2 more than 8. So '10 take away 8' can be solved by counting up 2, or the difference between 10 and 8 is 2.* This is a major cluster for first grade. The activities and strategies are both developmental and reinforcement.

Future Learning

In second grade, students will use addition and subtraction within 100 to solve problems involving situations of adding to, taking from, putting together, taking apart and comparing with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Additional Findings

According to *Adding It Up*, "for students in grades K–2, learning to see the part-whole relations in addition and subtraction situations is one of their most important accomplishments in arithmetic" (p. 191).

According to *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, "Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as 'counting on' two). They use properties of addition (commutatively and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., 'making tens') to solve addition and subtraction problems involving basic facts" (p. 13).

Grade 1 Mathematics, Quarter 3, Unit 3.5

Tens, Tens, Tens

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Understand that the two digits of a two-digit number represent amounts of tens and ones.
- Understand that multiples of ten are special cases meaning 10, 20, 30, and so on, and they refer to one, two, three tens and zero ones.
- Accurately use concrete models or drawings to subtract multiples of 10 within 90.
- Use the relationship between addition and subtraction as a strategy to accurately solve subtraction problems involving multiples of 10.
- Use strategies based on place value and the properties of operations to subtract multiples of 10.
- Record the strategy used to subtract multiples of 10 within 90 using a written method.
- Accurately explain the strategy for subtracting multiples of 10 and connect the written method to the strategy.
- When given a two-digit number, mentally find 10 more or 10 less than the number.

Mathematical practices to be integrated

Attend to precision.

- Use reasoning to explain the use of a group of some tens and zero ones to compose the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90.
- Attend to the meaning of quantities.
- Clearly define that a bundle of 10 ones is a ten.

Look for and express regularity in repeating reasoning.

- Notice that if a ten is added to a number, the numeral in the tens digit increases by one, but adds a quantity of 10 to the number.
- Look for and describe patterns in the number system.
- Look for shortcuts in solving equations.

Essential questions

- Can you explain how many tens are in a given number?
    - How can place value help you add a two-digit number to a one-digit number?
- What do you notice is happening to the numbers (10 to 20, 20 to 30, etc.)?
- How can place value help you add/subtract multiples of ten?
- What is ten less than _____ (a given number within 100)? What is ten more?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	1.NBT

Understand place value.

1. NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

*For example, students may explain their reasoning by saying that they have one more or one less ten than before. Drawings and layered cards can be used to connect with place value and can be used in the explanation.

1.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Common Core State Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students gained the understanding that the numbers 11–19 are composed of 10 ones and 1, 2, 3, 4, 5, 6, 7, 8, or 9 ones. In kindergarten, students represented addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps, acting out situations, verbal explanations, expressions, or equations). They added and subtracted fluently within 5. They also composed and decomposed numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and recorded each composition or decomposition by drawing or equation.

Current Learning

Students understand that the two digits of a two-digit number represent amounts of tens and ones. They understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 are special cases that refer to one, two, three, four, five, six, seven, eight, or nine tens and 0 ones. Students also subtract multiples of 10 in the range of 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; they relate the strategy to a written method and explain the reasoning used. This is a major cluster that will include introductory as well as developmental level activities. They mentally find ten more or ten less than a number and explain their reasoning. This is a critical area and includes introducing advanced strategies as well as developing previously learned strategies (refer to 1.OA.6).

Future Learning

In second grade, students will understand that 100 can be thought of as a bundle of 10 tens called a “hundred.” They will understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). They will be able to mentally subtract 10 or 100 from a given number 100–900. They will write an equation to express the total as a sum of equal addends; mentally add 10 or 100 to a given number 100–900; and mentally subtract 10 or 100 from a given number 100–900. They will explain why addition and subtraction strategies work, using place value and the properties of operations.

Additional Findings

“It is absolutely essential that students develop a solid understanding of the base ten numeration system and place value concepts by the end of grade 2.” (*Principles and Standards for School Mathematics*, p. 81)

“Research has shown that students can learn well from a variety of different instructional approaches, including those that use physical materials to represent hundreds, tens, and ones.” (*Adding It Up,* p. 197)

In *Progressions K–5, Number and Operations in Base Ten*, (p. 6), it is stated that grade 1 students use their base-ten work to help them recognize that the digit in the tens place is more important for determining the size of a two-digit number. They use this understanding to compare two two-digit numbers, indicating the result with the symbols >, =, and <.

Grade 1 Mathematics, Quarter 4, Unit 4.1

Addition and Subtraction within 100 – Multiple Strategies to Add/Subtract (12 Days)

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Represent and solve compare problems using object, drawings, and equations.
- Students use commutative property of addition to add but do not need to use formal terms (e.g., 8 + 3 = 11 is known then 3 + 8 = 11)
- Add and subtract within 20 using various strategies with fluency within 10.
- Add a two-digit and a one-digit number within 100 using concrete models or drawings and strategies to solve problems.
- Understand that in adding two-digit numbers, one adds tens and tens and ones and ones; and sometimes it is necessary to compose a ten.
- Develop strategies to solve compare problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Breaking apart and using prior knowledge to solve problems.
- Abstract a given situation and represent it symbolically.

Model with mathematics.

- Use objects, drawings, or equations to model and solve addition or subtraction situations.
- Analyze relationships to draw conclusions.
- Question if their prediction or solution makes sense.

Essential questions

- If the order of the addends change; will the sum change and how do you know?
- Can you explain the easiest and/or fastest way to solve this problem?
- When adding, what happens when you have more than 9 ones in the ones place?
- How can you represent this problem using objects and equation(s)?

Written Curriculum

Number and Operations in Base Ten	1.NBT

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Operations and Algebraic Thinking	1.OA

Understand and apply properties of operations and the relationship between addition and subtraction.

1. OA.3 Apply properties of operations as strategies to add and subtract.[3] Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

[3] Students need not use formal terms for these properties.

Add and subtract within 20.

1. OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Common Core State Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten, students added and subtracted within 10, e.g., by using objects or drawings to represent the problem. They used objects or drawings to record equations, e.g., 5 = 2 + 3 and 5 = 4 + 1). They fluently added and subtracted within 5. They also composed and decomposed numbers from 11 to 19 into ten ones and some further ones.

Current Learning

In first grade, students apply properties of operations as strategies to add and subtract. Students do not need to use formal terms for these properties. They use a variety of strategies to add and subtract within 20 and demonstrate fluency within 10. They will also add within 100, including adding a two-digit number and a one-digit number using various strategies. These are all major clusters and will include introductory as well as developmental activities. Students are introduced to compare problems in this unit, and instruction should be at the developmental level for these difficult problem types. (See CCSS Glossary, Table 1).

Future Learning

In second grade, students will fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. They will fluently add within 20 using mental strategies. They will know from memory all sums of two one-digit numbers.

Additional Findings

Leaning occurs best through dialogue, discussion, and interaction. Learners must be actively involved in the process. A variety of models must be used to meet the needs of all learners. Learners benefit from reviewing, critiquing and revising one another's work. (*A Research Companion to Principles and Standards for School Mathematics*, p. 48)

Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition. (*Progressions*, p. 360)

Grade 1 Mathematics, Quarter 4, Unit 4.2

Comparing Values of 2-Digit Numbers Using Appropriate Symbols (<, >, =) (9 Days)

Overview

Number of instructional days: 9 (1 day = 45–60 minutes)

Content to be learned

- Compare two two-digit numbers.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Use place value to determine the value of two two-digit numbers.
- Use symbols to show the value of a two-digit number.

- Represent the problem with visuals or math tools.
- Reflect on their thinking by visualizing the problem.

Attend to precision.

- Talk with other students using correct mathematical vocabulary.
- Recognize greater than, less than, and equal situations.

Essential questions

- How can you use your understanding of tens and ones to compare two 2-digit numbers?
- How do you know which symbol to use to show the value of a number?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**1.NBT**

Understand place value.

1. NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

* Tasks should focus on the understanding that the digit in the “tens” place is more important for determining the size of a two-digit number.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Prior Learning

In kindergarten, students composed and decomposed numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and recorded each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8).

Current Learning

In first grade, quarters 1 and 2, students were introduced to place value. In this unit, students compare two 2-digit numbers based on meaning of the tens and ones digits. This is a major cluster that should include introductory as well as developmental activities.

Future Learning

In second grade, students will compare two 3-digit numbers based on meanings of the hundreds, tens, and ones digits.

Additional Findings

As stated in *Principles and Standards for School Mathematics*:

"Understanding of number develops in prekindergarten through grade 2 as children count and learn to recognize 'how many' in sets of objects. A key idea is that a number can be decomposed and thought about in many ways. For instance, 24 is 2 tens and 4 ones and also two sets of 12. Making a transition from viewing 'ten' as simply the accumulation of 10 ones to seeing it both as 10 ones and as one 10 is an important first step for students toward understanding the structure of the base-ten number system" (Cobb and Wheatley 1988) (p. 33).

As stated in the *Progressions*, in a compare situation, two quantities are compared to find 'how many more' or 'how may less.' One reason these types of problems are more advanced than the other two major types is that, in compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation (p. 12).

Grade 1 Mathematics, Quarter 4, Unit 4.3

Counting on to Add and Subtract (10 Days)

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Count on by two to add 2 when solving addition problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Represent the problem with visuals or

- Count back 2 to subtract 2 when solving subtraction problems.

math tools.

- Reflect on their thinking by visualizing the problem.
- Recognize and use patterns or properties of operations to solve problems.

Attend to precision.

- Talk with other students using correct mathematical vocabulary.
- Focus on clarity and accuracy of problem solving.

Essential questions

- How does counting help you solve an addition problem?
- How does counting on help you solve an addition problem?
- How does counting back help you solve a subtraction problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**1.OA**

Add and subtract within 20.

1. OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results

Clarifying the Standards

Prior Learning

In kindergarten, students fluently added and subtracted within 5.

Current Learning

In first grade, quarters 1 and 2, students were introduced to addition and subtraction within 10. In this unit, students relate counting to addition by counting on two to add 2. They relate counting to subtraction by counting back 2 to subtract 2. This is a major cluster that should include introductory as well as developmental activities.

Future Learning

In second grade, students will determine whether a group of objects, up to 20, has an odd or even number of members, by pairing objects or counting them by 2s.

Additional Findings

As stated in *Principles and Standards for School Mathematics*:

“Understanding of number develops in prekindergarten through grade 2 as children count and learn to recognize ‘how many’ in sets of objects. A key idea is that a number can be decomposed and thought about in many ways. (Cobb and Wheatley 1988) (p. 33).

Grade 1 Mathematics, Quarter 4, Unit 4.4

Organizing, Representing, and Analyzing Data With Up to Three Categories (8 Days)

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Accurately organize data with up to three categories.
- Accurately represent data with up to three categories.
- Accurately interpret data with up to three categories.
- Ask and answer questions about the total number of data points and how many are in each category.
- Accurately compare how many more or less are in one category than in another.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Develop arguments using objects or drawings from data representations.
- Clearly communicate conclusions to others.
- Justify conclusions from data analysis.
- Use questions to clarify or improve the arguments of others.

Model with mathematics.

- Apply addition or subtraction strategies to compare data.
- Draw conclusions based on data.
- Analyze relationships between categories.

Essential questions

- Why is it important to organize data?
- How can you represent data?
- What can you tell me about this data?
- How can you tell if there is more or less in a category?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	1.MD

Represent and interpret data.

1. MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students have classified objects into given categories. They have counted the numbers of objects (less than or equal to 10) in each category and sorted the categories by count.

Current Learning

Students organize, represent, and interpret data with up to three categories. They ask and answer questions about total number of data points, how many in each category, and how many more or less are in one category than in another. This is a supporting cluster and includes developmental activities.

Future Learning

In second grade, students will draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. They will solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Additional Findings

"Children strengthen their sense of number by solving problems involving measurements and data. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships." (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, p. 13)

"Through their data investigations, young students should develop the idea that data charts and graphs give information." *(Principles and Standards for School Mathematics*, p. 113)

Grade 1 Mathematics, Quarter 4, Unit 4.5

Decomposing Shapes into Halves and Fourths Using Appropriate Vocabulary; Using Attributes to Compose New Shapes

Overview

Number of instructional days: 7 (1 day = 45–60 minutes)

- **Content to be learned**
- Identify a shape using defining attributes (closed, number of sides, length of sides).
- Identify non-defining attributes of a shape (size, color, orientation).
- Build and draw shapes to possess defining attributes.

- Compose and decompose two-dimensional shapes.
- Compose and decompose three-dimensional shapes.
- Partition circles and rectangles into two and four equal shares.
- Use the words halves, fourths, and quarters to describe the equal shares.
- Use half of, fourth of, and quarter of to describe the equal shares.
- Understand that a whole is made up of two halves or four fourths.
- Understand that decomposing into more equal shares causes each share to be smaller.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use appropriate tools to develop a deeper understanding of shape attributes.
- Select appropriate tools to determine defining attributes.

Model with mathematics.

- Take risks and make predictions.
- Use a variety of math tools.

Look for and make use of structure.

- Look closely to discern the structure of shapes.
- Look for patterns in the number of sides, attributes of shapes, sided lengths, etc.

Essential questions

- How do you know if a shape is two-dimensional or three-dimensional?
- What new shapes can you make by putting together these shapes? Show me another shape you can make from your new shape?
- How do you know if an equal share of a whole is a half or a fourth (quarter)?
- If you partition a circle or rectangle into two equal parts, what is one part called?
- If you partition a circle or rectangle into four equal parts, what is one part called?
- How many fourths/ halves do you need to make a whole?
- Explain how you can partition a rectangle

- How can you partition a circle (or rectangle) into two (or four) equal parts?
- How can you partition a circle (or rectangle) into two (or four) equal shares? How do you know your shares are equal?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**1.G**

Reason with shapes and their attributes.

1. G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1. G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]

[4] Students do not need to learn formal names such as "right rectangular prism."

1. G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of, fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students identified shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). They analyzed and compared two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Current Learning

In first grade, students will distinguish between defining attributes (e.g., triangles are closed and three-sided) and non-defining attributes (e.g., color, orientation, overall size). They will build and draw shapes to possess defining attributes. They will compose two- or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.

As noted in the footnote, students do not need to learn formal names such as "right rectangular prism." Students partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of. They describe the whole as two of or four of the shares. They understand for these examples that decomposing into more equal shares creates smaller shares.

Future Learning

In second grade, students will partition a rectangle into rows and columns of same-size squares and count to find the total number. They will also partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, four fourths. They will also recognize that equal shares of identical wholes need not have the same shape.

In third grade, students will be introduced to mathematical notation for fractional quantities (1/2, 2/3, 2/4, etc.).